SUSTAINABLE DEVELOPMENT VISION IS A DYNAMIC CONCEPT

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ABSTRACT

Sustainable development has emerged as a fundamental framework for evaluating the progress and advancement of societies. In recent decades, growing attention has been directed toward its multidimensional nature, encompassing economic, educational, and environmental aspects. However, much of the existing literature has examined these dimensions in isolation, leaving a gap in understanding their dynamic interrelations and practical implications. This study seeks to address that gap by employing mathematical modelling and simulation techniques to analyse the interconnected dynamics of sustainable development. By designing and testing models that reflect community realities, the research offers a novel perspective on the mechanisms required for societies to achieve sustainable growth. Such an approach contributes not only to theoretical advancement but also provides practical insights into building sustainable cities that integrate diverse sources of development. Furthermore, the study positions itself as an applied field investigation, bridging theory and practice through quantitative modelling. A comparative analysis with previous research highlights both continuity and innovation, underscoring the study's contribution to the broader discourse on sustainable development. Ultimately, the findings aim to enrich academic understanding while offering actionable strategies for policymakers and institutions striving toward sustainability.

Keywords: sustainability – environmental - communities - modelling

الخلاصة

لقد برز مفهوم التنمية المستدامة بوصفه إطاراً أساسيًا لتقييم تقدم المجتمعات وازدهارها. وفي العقود الأخيرة، تزايد الاهتمام بطبيعتها متعددة الأبعاد، التي تشمل الجوانب الاقتصادية والتعليمية والبيئية. غير أن معظم الدراسات السابقة تناولت هذه الأبعاد بصورة منفصلة، مما ترك فجوة في فهم ترابطها الديناميكي وانعكاساتها العملية. ويسعى هذا البحث إلى سد تلك الفجوة من خلال توظيف تقنيات النمذجة الرياضية والمحاكاة لتحليل ديناميكيات التنمية المستدامة المترابطة. ومن خلال تصميم نماذج واختبارها بما يعكس واقع المجتمعات، يقدّم البحث منظوراً جديداً حول الآليات اللازمة لتحقيق النمو المستدام. ولا يقتصر هذا النهج على الإسهام في التطور النظري فحسب، بل يوفّر أيضًا رؤى عملية لبناء مدن مستدامة قادرة على دمج مصادر التنمية المتنوعة. علاوة على ذلك، يضع البحث نفسه في إطار دراسة ميدانية تطبيقية، تربط بين النظرية والممارسة عبر النمذجة الكمية. وتُظهر المقارنة مع الدراسات السابقة عناصر الاستمرارية والابتكار معًا، مما يبرز إسهام هذا العمل في الخطاب الأكاديمي الأوسع حول التنمية المستدامة. وفي نهاية المطاف، تهدف النتائج إلى إثراء الفهم العلمي وتقديم استراتيجيات عملية لصنّاع القرار والمؤسسات الساعية إلى تحقيق الاستدامة

1 . Introduction

Mathematical modelling is a fundamental tool in advancing sustainable development, as it enables the systematic analysis of complex interactions between energy systems, economic growth, and environmental constraints. By formulating equations that capture the dynamics of renewable and non-renewable energy, resource consumption, and efficiency, mathematical models provide a structured framework to evaluate sustainability indices and predict long-term outcomes. Thus, mathematical modeling serves not only as a scientific method but also as a strategic instrument for guiding policies, optimizing energy systems, and ensuring that development pathways remain aligned with ecological balance and social justice. The most important mathematical models that studies focus on (Optimization Models- Eco-Economic Models- Dynamic Differential Models- Green Mathematics Models).[1] Development is a fundamental pillar of stability and progress across the economic, social, and educational domains, and it is directly reflected in human advancement. It represents a continuous process—whether comprehensive or partial—that adopts various forms aimed at elevating the human condition and achieving well-being in a manner consistent with societal needs and capacities. As a planned activity, development seeks to bring about changes in individuals and communities in terms of knowledge and experience, performance and methods of work, as well as attitudes and behavior. This enables members of society to fulfill their roles efficiently and with high productivity. [2] Sustainable development extends this concept by incorporating the environmental dimension alongside the economic, social, and educational aspects, ensuring the proper investment of available resources to meet present needs while safeguarding the rights of future generations. For more than two decades, the world has faced the threat of environmental deterioration, which must be addressed without neglecting the requirements of economic growth, equality, and social justice. [3] Sustainable development requires creating suitable living conditions for all individuals without exceeding the Earth's carrying capacity, and it promotes balanced consumption that avoids excessive reliance on natural resources. It can be pursued in two directions: either through comprehensive, integrated, and harmonious development, or through progress in one of the main fields in isolation from others—such as the economic, political, or social domains—or in subfields such as industry, agriculture, or tourism. In essence, sustainable development represents a process of economic and social transformation aimed at moving societies toward improved conditions in line with their needs and capacities, while safeguarding the environment, human heritage, rights, and collective gains. It involves implementing medium- or long-term plans that elevate economic, humanitarian, and environmental conditions without causing harm to natural or social systems. Integrated development, in particular, enhances opportunities for some members of society without diminishing those of others. It results in tangible increases in production and services that are comprehensive and interconnected with societal dynamics. This process relies on modern scientific methods in technology, organization, and management to ensure balanced progress and sustainability. [4] Global efforts to achieve sustainable development are pivotal in addressing the environmental, social, and economic challenges facing humanity. At the international level, several measures have been undertaken to promote sustainability. Among the most significant is the 2030 Agenda for Sustainable Development, adopted by the United Nations in 2015. This agenda introduced 17 Sustainable Development Goals (SDGs), covering a wide range of issues including poverty, hunger, education, water, energy, and climate change. Together, these goals provide a comprehensive framework to guide global action toward achieving sustainable development by 2030 . [5-6]. One of the most important recent studies is Education for Sustainable Development (ESD) through mathematical modeling. A quasi-experimental study (2026) with 68 pre-service primary school teachers. It addressed sustainability-related modeling activities such as water consumption, waste management, sustainable transportation, and energy use. Results: A significant improvement in teachers' confidence in designing sustainability-based mathematics lessons, and modeling was considered an effective pedagogical approach for integrating sustainability issues into education. Covering a wide range of issues such as poverty, hunger, education, water, energy, and climate change, the 2030 Agenda for Sustainable Development aims to direct global efforts toward achieving sustainability by 2030. Adopted by the United Nations in 2015, this agenda introduced 17 Sustainable Development Goals (SDGs) that serve as a comprehensive framework for international action. In 2024, a study was introduced that focused on designing a mathematical model for renewable energy. [7] Integrated policy models were presented to achieve synchronized progress on SDG goals by simulating the interactions of health, economic and environmental policies. In [8] The models helped develop clearer concepts of the relationship between humans and nature . To facilitate hypothesis testing and scenario comparison .Integrating the social dimension: The study emphasized the importance of incorporating power disparities and social justice into the models. [9] In this research, the most influential factors in dynamic development were analyzed, offering new insights into the interplay between energy systems and sustainable growth.

2. dimension of development: [10-12]

Sustainable development is a comprehensive framework that seeks to balance the current needs of humanity with the rights of future generations. To achieve this balance, three main dimensions are integrated: economic, social, and environmental:

1. The material (economic)

The economic dimension focuses on meeting human needs through the production and consumption of resources, eliminating poverty, and generating modern means of growth. It emphasizes the use of technology and science to enhance productivity while maintaining capital.

. Environmental dimension 2

The **environmental dimension** highlights the limits of resource consumption and the necessity of preserving ecological balance. Housing projects, population growth, water use, soil conservation, forestry, and fisheries must all be managed within sustainable boundaries to prevent depletion and ensure long-term stability. Together, these dimensions illustrate that sustainable development is not merely about short-term prosperity but about creating systems that endure over time. It requires integrated planning that elevates living standards without exceeding the Earth's carrying capacity, while safeguarding natural resources and ensuring social justice.

Table (1) Compare between (Economic- Environment) Dimension

Aspect	Economic Dimension	Environment Dimension
Growth and prosperity	Sustainability and balance	Primary goal
Short –to medium term	Long –term (future focus	Time Horizon
Financial indicators	Ecological indicators	Measure ment
Inequality ,inflation, market	Climate change ,resource depletions'	Risks

Figure (1): Show dimensions of sustainable development

Renewable energy3.

In this section, the basic concepts of renewable and non-renewable energy are presented, along with their relationship to the dimensions of sustainable development (economic and environmental). The discussion is supported by mathematical modeling that incorporates the key factors of renewable energy.

1.**Renewable energies**

Renewable energy refers to sources that are naturally generated and continuously replenished. These resources are characterized by being inexhaustible, widely available in nature, and environmentally clean. Their use does not result in pollution, making them essential for sustainable growth. Examples include solar, wind, and hydropower.

2.Non-renewable energy

Non-renewable energy is derived from natural resources that are finite and depleted when consumed. These sources—such as coal, oil, and natural gas—were formed over millions of years and exist in limited reserves. Once exhausted, they cannot be replenished within a short period, making their use unsustainable in the long term.

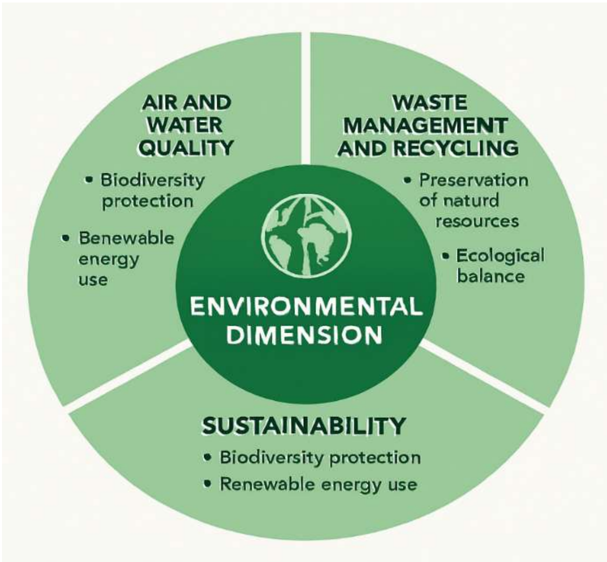

Figure (2): Factors affecting on dimensions of sustainable development

Table (2) classification Environment Dimension on Environmental dimension

Environmental dimension	
Nonrenewable energy It takes a long time to make up for it	Renewable energy Not enforceable
Gas- Oil- Coal	Wind energy- the sun energy- Water energy- Underground energy

4. Mathematical analysis of development dimensions

Mathematical analysis of development dimensions provides a methodological framework for understanding the complex relationships between economic and environmental variables. Mathematical models, differential equations, and statistical analysis methods are used to characterize development dynamics and measure the balance between economic growth and environmental sustainability.

1. Mathematical modeling of the energy clean equation

The concept of clean energy modeling is based on integrating renewable and non-renewable energy sources within a mathematical framework that reflects both efficiency and sustainability. Clean energy is defined as the net output of renewable sources (such as solar, wind, and hydropower) adjusted by conversion efficiency, while accounting for the environmental costs of non-renewable sources (such as coal and gas). Mathematical analysis of development dimensions provides a methodological framework for understanding the complex relationships between economic and environmental variables. Mathematical models, differential equations, and statistical analysis methods are used to characterize development dynamics and measure the balance between economic growth and environmental sustainability. Present the mathematical model.

$$\frac{dC_e(t)}{dN} = E_c(t)\, R_e(t)+W_e\,.\,\eta(t) - {N_r}^2\beta\,\frac{\delta}{N_r(t)}\,.\,CI(t) + \sqrt{S_e}\ (1.1)$$

$$\frac{dN_r(t)}{dN} = \frac{3*\beta}{\sqrt{S_e}} \propto \delta\ (1.2)$$

Where $C_e(t)$Energy generated by time, $R_e(t)$ be renewable energy , $N_r(t)$ be nonrenewable energy , $\eta(t)$ Dynamic efficiency coefficient , $CI(t)$ Carbon intensity factor for non-renewable energy , $E_c(t)$ capacity factor , α be Conversion efficiency We Wind energy, Se sun energy , Co be Coal ,and δ and β are constant.

Using the integration of nonrenewable energy equation (1.2) for N, using the boundary condition of the coefficient of the nonrenewable energy $N_r(0) = 0$ obtained by completing the solution.

$$N_r(t) = \frac{3 * \beta}{\sqrt{S_e}} \propto \delta\, N_r + A \ (1.3)$$

Where's A be integration constant. Now, by applying the Boundary conditions for the the coefficient of the nonrenewable energy. $N_r(0)=0$ it is getting the integration constant . $A=0$, Substituting for A into the equation (1.3) ,to get the general form :

$$N_r(t) = \frac{3 * \beta}{\sqrt{S_e}} \propto \delta\, N_r \ (1.4)$$

Substituting nonrenewable energy equation (1.4) into clean energy equation (1.1) given:

$$\frac{dC_e(t)}{dN} = E_c(t)R_e(t) + W_e\,.\,\eta(t) - N_r\frac{\sqrt{S_e}}{3 * \propto \delta}.\,CI(t) + \sqrt{S_e}\ (1.5)$$

To obtain the general solution to the equation, we integrate clean energy equation (1.5) for N

$$C_e(t) = E_c(t)R_e(t)N + W_e\,.\,\eta(t)N - N_r\frac{\sqrt{S_e}}{3 * \propto \delta}.\,CI(t)N + \sqrt{S_e}\,N + B\ (1.6)$$

Where B be integration constant. Now, by applying The Boundary conditions for the the coefficient of the nonrenewable energy. $C_e(0)=0$, $E_c(t) = 0\,,\eta(t) = 0\,, CI(t) = then$ integration constant . $B=-\sqrt{S_e}\,N$ Substituting for B into the equation (1.6) ,to get the general form of clean energy equation :

$$C_e(t) = E_c(t)R_e(t)N + W_e\,.\,\eta(t)N - N_r\frac{\sqrt{S_e}}{3 * \propto \delta}.\,CI(t)N\ (1.7)$$

The equation above represents the final formula for the energy equation resulting from a change in time.

2. The Dynamic Equation of Sustainability

This equation. Based on the relationship between economic growth and resource consumption, and its relationship to the Gini coefficient, which measures the degree of income inequality. The equation is represented: as follows

$$\frac{ds}{dN} = \frac{GDP}{R} + G + N_r(t) \ (1.8)$$

Table (3) classification Environment Dimension on Environmental dimension

Symbol	Interpretation
S	The Sustainable Development Index (SDI) is a measure of how efficiently countries achieve human development while staying within ecological limits.
R	The rate of nature consumption refers to the speed at which humans use natural resources compared to the Earth's ability to regenerate them.
G	The Gini coefficient is a statistical measure used to represent inequality within a distribution, most commonly applied to income or wealth
GDP	gross domestic product

Using the integration of Dynamic Equation of Sustainability (1.8) for N, using the boundary condition of the coefficient of economic the was obtained by completing the solution.

$$S = \left(\frac{GDP}{R} + G + N_r(t)\right) N + D \ (1.9)$$

Where D be integration constant. Now, by applying The Boundary conditions for the coefficient of the nonrenewable energy. S(0)=0 it is getting the integration constant D =0 , Substituting for A into the equation (1.9) ,to get the general form :

$$S = \left(\frac{GDP}{R} + G + N_r(t)\right) N \ (1.10)$$

The equation above represents the dynamic equation of development, which plays an important role in studying dimensions.

5. Results and discussion

Moroccan Journal for Publishing Scientific Research (ISSN : 3085-4873) – Issue 20 – August 2026

This section introduces the factors influencing sustainable development and explains in detail the relationship between carrying capacity and conversion efficiency with the

Sustainable Development Index (SDI). The results were analyzed using the Mathematica program.

1. **Carrying capacity** Carrying capacity is a metric that reveals the strengths and weaknesses of each type of energy source and determines how they can be combined to achieve a sustainable and secure energy system. Figure (3) illustrates the relationship between carrying capacity and non-renewable energy. The figure shows that oil has the highest carrying capacity compared to gas and coal during the period 2023–2025, reflecting the continued heavy reliance on oil as a primary energy source. It also shows that all three sources are experiencing a gradual increase in carrying capacity over time, indicating an expansion in the exploitation of non-renewable resources. This trend reveals a clear gap between the sources, with coal remaining the least sustainable, while oil dominates the energy landscape. Theoretically, this chart reflects that current energy policies focus on boosting production from conventional sources for short-term economic gains, but this places increasing environmental pressures and threatens long-term sustainability. Therefore, the analysis underscores the need to restructure energy policies by promoting investment in renewable energy and improving energy conversion efficiency, thereby reducing the gap between economic growth and environmental protection. Figure (4) illustrates the relationship between carrying capacity and renewable energy. In this case, the carrying capacity appears lower and more unstable, since its dependence on network integration leads to natural fluctuations.

conversion efficiency2 .2

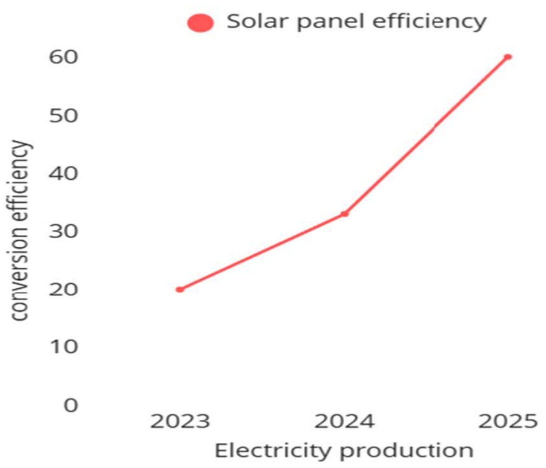

Figure (5) Show that conversion efficiency and electricity production

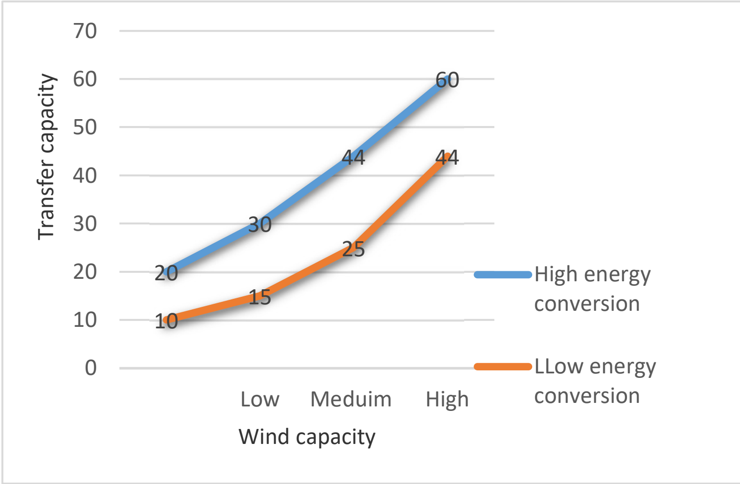

Figure (6) Show that conversion efficiency and Wind capacity

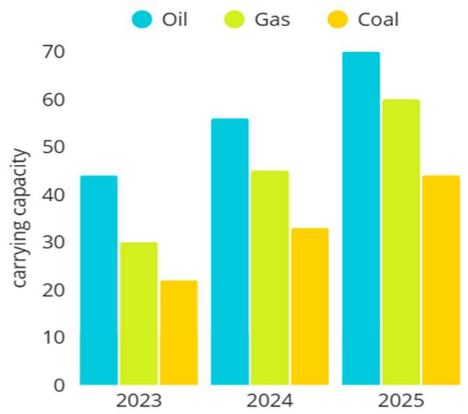

Figure (3) shows carrying capacity in non-renewable

Energy -

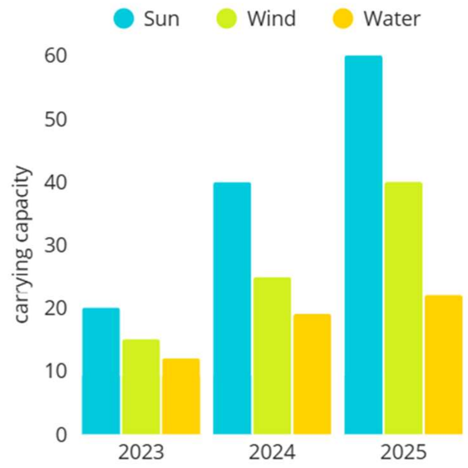

Figure (4) shows carrying capacity in renewable Energy

3. Sustainability index

Figure (7) illustrates the relationship between the rate of natural resource consumption and the sustainability index. An increasing rate of resource consumption hurts sustainability, reducing the overall index value. Imbalance: The rate of natural resource consumption exceeds the Sustainable Development Goals (SDGs), reflecting a clear gap between resource consumption and actual sustainability. Increasing Environmental Pressure: The high rate of consumption (more than half) indicates that economic and social activities are heavily reliant on natural resources, putting pressure on the environment and threatening the sustainability of future generations Shortcomings in Sustainable Development Policies: The shortfall in SDG targets compared to consumption indicates the inadequacy or ineffectiveness of efforts to balance development and the environment. The Need for Restructuring: This imbalance highlights the need to strengthen resource management policies, increase investment in renewable energy, and improve resource efficiency to bridge the gap between consumption and sustainable development. Figure (8) illustrates the relationship between production rates and the sustainability index. High production rates without consideration of development resources negatively affect the sustainability index, whereas efficient production helps maintain balance. Figure (9) illustrates the relationship between environmental impact rate and GDP. As GDP increases, the environmental growth rate is negatively affected, highlighting the trade-off between economic expansion and ecological sustainability.

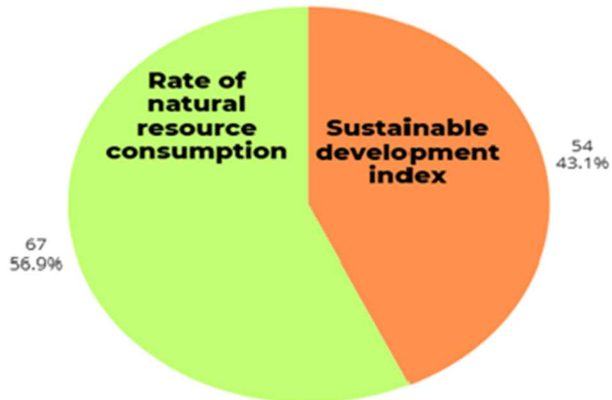

Figure (7 ... d sustainability index

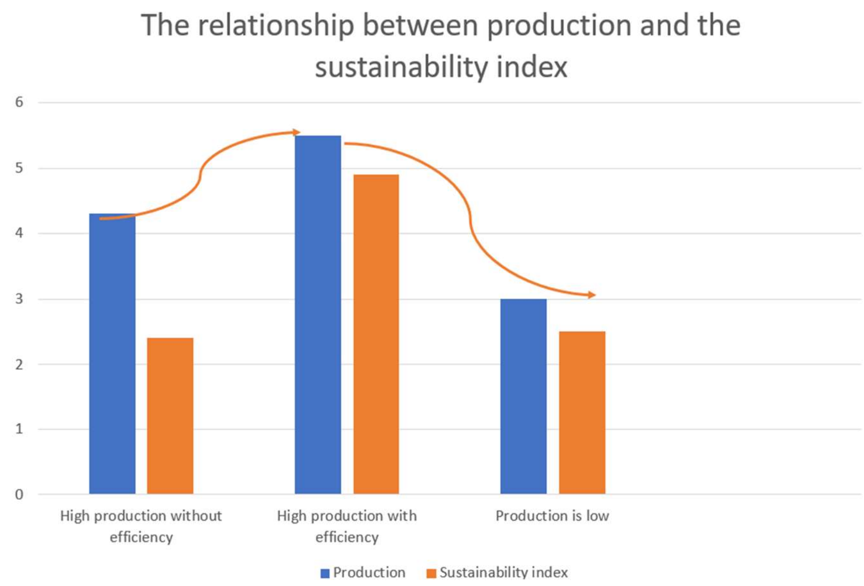

Figure (8) :

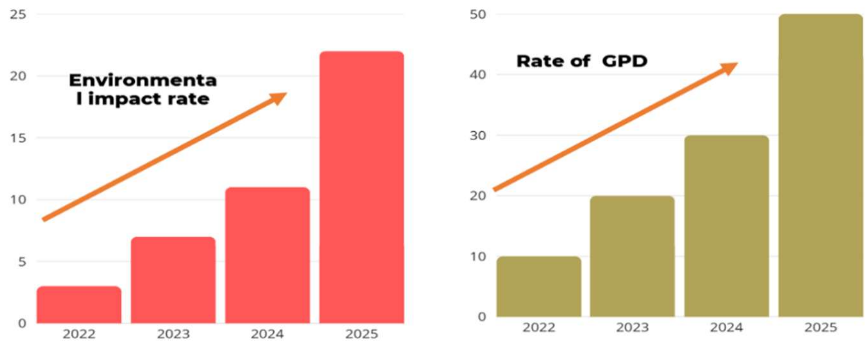

Figure (9) Show that enrivnmenta impact rate with GDP

Table (4): Comparison between Environment a impact rate and rate of GPD

Environment a impact rate				Rate of GPD			
2022	0.1	2	4	2	5	7	10
2023	1	3	7	4	10	15	20
2024	3	5	12	6	12	22	30
2025	7	12	22	10	25	40	50

The figure (10) illustrates that the sustainability index (S) is determined by the balance between three key elements: the stock (R) of natural resources, the outflow (C) representing the rate of consumption or depletion, and the inflow (G) reflecting the rate of natural regeneration or recycling. If the rate of regeneration exceeds the rate of consumption, the sustainability index rises, reflecting the community's ability to maintain its resources over time. Conversely, if consumption exceeds the rate of regeneration, the index falls, indicating an imbalance that leads to resource depletion and environmental degradation. Therefore, sustainability is not a static state but a dynamic process influenced by changes in consumption and regeneration rates, making the (S) index an important explanatory tool for understanding the relationship between humanity and nature. This analysis highlights that achieving sustainability requires effective resource management policies, promoting recycling, and relying on renewable energy sources to ensure that the inflow remains equal to or exceeds the outflow.

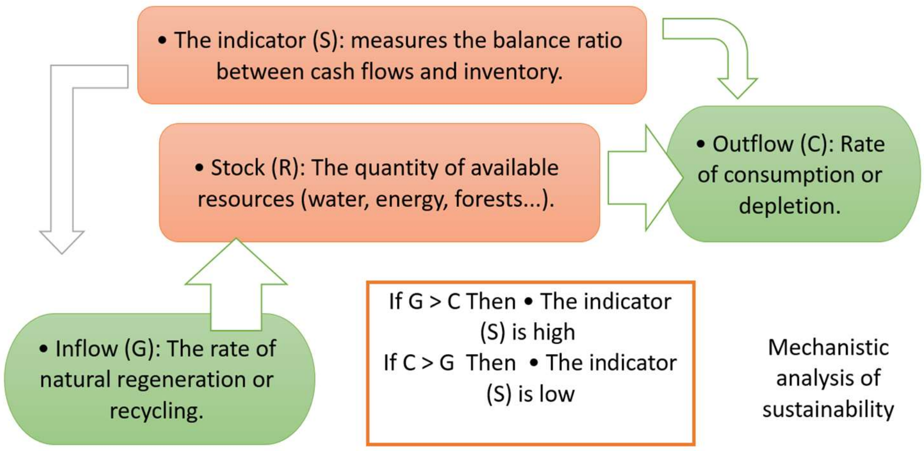

Figure (10) shows the mechanics analysis of sustainability

Table (5): Reviews of previous studi

2023	2025	2026
The most important finding of the Modeling Sustainability Across Sectors study published in the PNAS Journal is that it has established a practical framework for developing dynamic models that link natural, social, and economic systems, with a focus on integrating the human dimension and equity into sustainable policies.	The System Dynamics Modelling for Sustainable Energy Planning study was published in the journal Energy Policy. Focused on the use of dynamic modelling for renewable energy planning, and showed that dynamic models help to reduce costs, improve efficiency, and evaluate long-term policies to achieve sustainability goals.	The study "AI-driven Sustainability Models in Developing Countries," published in the IEEE Transactions on Systems, Man, and Cybernetics: Systems journal, focused on the role of artificial intelligence in supporting sustainable development in developing countries.

6. Conclusions

Based on the analysis and findings, the following conclusions highlight the relationship between energy sources, production, and the environmental index. These points emphasize the role of efficiency, recycling, and conversion capacity in fostering sustainable development, while also addressing the challenges posed by excessive consumption of natural resources.

Carrying Capacity of Energy Sources

The carrying capacity is higher for non-renewable energy sources, while it decreases for renewable energy sources due to environmental conditions. This variation reflects the nature of the resources used and their sensitivity to external factors.

Production and Efficiency

High production combined with efficiency fosters a sustainable environment. The relationship between production and the environmental index is direct: as production increases, environmental pressure also rises. This highlights the importance of managing resources wisely to balance growth and sustainability.

Recycling and Consumption

Greater production and recycling enhance the environmental index, whereas excessive consumption of natural resources reduces it. Recycling therefore plays a crucial role in maintaining equilibrium between production and environmental preservation.

Conversion Capacity in Renewable Energy

Conversion capacity is closely related to electricity generation from solar energy, which depends on the efficiency of solar panels. It is also linked to wind power, where higher turbine capacity plays a major role in transforming wind energy into electricity.

Applied Summary

The study provides quantitative tools that can be directly utilized in Iraq for managing water and energy resources, developing urban areas, and modernizing education. These applications transform the research from a purely theoretical framework into a practical instrument that assists decision-makers and institutions in addressing local challenges and achieving sustainable development.

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